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STEPWISE LINEAR REGRESSION PARTICULAR INTEGRALS FOR UNCOUPLED THERMOELASTICITY WITH BOUNDARY ELEMENTS

SUNIL SAIGAL
Department of Civil Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A.

and

A. GUPTA and J. CHENG
Mechanical Engineering Department, Worcester Polytechnic Institute, Worcester, MA 01609, U.S.A.

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Abstract—This paper presents a simple technique for the efficient treatment of non-uniform body forces in boundary elements without the need for domain discretization and additional surface integral evaluations. The distribution of the non-uniform forces in the domain is described using regression polynomials. Particular integrals corresponding to these polynomials are developed for the solution of the non-homogeneous differential equations of elasticity. Two-dimensional and axisymmetric boundary element results for steady state and transient thermoelastic deformations are obtained using this formulation. Regression polynomials based on only the boundary points provide good results. Improved results are obtained by including internal points in the regression analysis. The method allows the use of low order polynomials to model a general distribution of thermal effects in the domain. Numerical data are given to provide comparisons of low order versus high order regression polynomials, and boundary only data versus combined boundary and domain data based regression polynomials.

INTRODUCTION

The boundary element method primarily involves the discretization of only the boundary of the object for a linear elastic analysis. The initial attempts to include the effects of body forces, see for example the textbook by Banerjee and Butterfield (1981), required the discretization of the domain into cells. Cruse *et al.* (1977) and Rizzo and Shippy (1978) presented a formulation that employs the body force potential and the divergence theorem to reduce the volume integrals involved into an equivalent surface integral for conservative body forces. Henry *et al.* (1987) and Pape and Banerjee (1987) introduced a technique based on the particular integral method for the solution of non-homogeneous ordinary differential equations to account for conservative body forces through direct nodal calculations. This technique avoided the evaluation of both volume integrals and surface integrals. Recently, Henry and Banerjee (1988a) developed a particular integral formulation for non-conservative body forces and presented the results for thermoelastic stress analysis. In this formulation, the temperature distribution in each region of a multi-region boundary discretization, is represented using global shape functions. The coefficients of the global shape functions are determined based on the temperatures at boundary element nodes and some interior points. The number of terms in these polynomials is equal to the number of locations whose temperatures were used in determining these coefficients. The particular integrals are then developed independently for each sub-region using the global shape functions. This technique has also been extended by Henry and Banerjee (1988b) for the elastoplastic analysis of two-dimensional and three-dimensional objects.

A different approach, also based on the particular integral method for the treatment of non-conservative body forces, is presented in this paper. This approach starts by representing the distribution of the body forces in a domain using polynomials that are based on the statistical procedure of linear regression. The process of linear regression forces the interpolation error to be normal to the domain in a least squared sense, for example, see the textbook by Neter *et al.* (1985). This results in a more accurate representation of the body force field compared to other available interpolations. Also, any general distribution can be approximated using a polynomial which retains the minimum number of terms required for an accurate interpolation. The number of terms retained does not have to be equal to the number of data points used to generate the regression polynomial. For example, a regression polynomial with only two terms but based on 16 data points provided excellent results for the solution of an axisymmetric cylinder under steady-state thermal loading as shown in a later section. After obtaining the regression polynomials representing a general distribution of body forces, particular integrals corresponding to these polynomials are presented. The regression particular integral formulation is implemented in a multi-zone boundary element program for two-dimensional and axisymmetric analysis, respectively. This approach is applied to the solution of steady-state thermoelastic and transient thermoelastic examples. The results obtained indicate good accuracy for these analyses. Numerical data are provided to study the effect of number of data points used to obtain the regression polynomial on the accuracy of thermoelastic responses. The results obtained by using different orders of regression polynomials are also compared.

THERMOELASTIC BEM WITH PARTICULAR INTEGRALS

The linear, non-homogeneous differential equation of equilibrium for an elastic body is given as

$$\zeta(u_i) = f_i \tag{1}$$

where

$$\zeta = (\lambda+\mu)u_{j,ji}+\mu u_{i,jj}, \tag{2}$$

λ,μ are Lame's constants, u_i is the displacement and f_i is the body force. For a body subjected to thermal loading, the contribution of this loading to the body force is given as

$$f_i^{(T)} = \alpha(3\lambda+2\mu)T_{,i} \tag{3}$$

where α is the coefficient of thermal expansion and T is the temperature. The particular integral method proceeds by first decomposing the response variables as

$$\begin{aligned} u_i &= u_i^c+u_i^p \\ t_i &= t_i^c+t_i^p \end{aligned} \tag{4}$$

where the superscripts c and p refer to the complementary and the particular solution, respectively. Thus, for thermoelastic displacements, u_i^c and u_i^p are given by the equations

$$\zeta(u_i^c) = 0 \tag{5}$$

and

$$\zeta(u_i^p) = \alpha(3\lambda+2\mu)T_{,i}. \tag{6}$$

Following the theory of linear non-homogeneous differential equations, it is known that the particular integral solution u_i^p is not unique and any expression satisfying eqn (6) is a valid particular integral. However, as shown in a later section, the particular integral solution must be chosen carefully for computational purposes.

For the boundary element method, the integral equation due to Somigliana satisfies the homogeneous eqn (5) and is used to describe the complementary solution as

$$C_{ij}(\xi)u_i^c(\xi) = \int_\Gamma [G_{ij}(x,\xi)t_i^c(x) - F_{ij}(x,\xi)u_i^c(x)]\, \mathrm{d}\Gamma(x) \tag{7}$$

where G_{ij} and F_{ij} are the fundamental solutions for displacements and tractions, respectively; C_{ij} is the corner tensor; and Γ is the boundary of the object. A procedure for determining the particular integral function u_i^p for the thermoelastic problem is described in the next section.

Discretizing the integral eqn (7) using boundary elements leads to a system of matrix equations given as

$$[F]\{u^c\} = [G]\{t^c\}. \tag{8}$$

Substituting for $\{u^c\}$ and $\{t^c\}$ from eqn (4) leads to the equation

$$[F](\{u\}-\{u^p\}) = [G](\{t\}-\{t^p\}). \tag{9}$$

$\{t\}$ is the traction obtained from the derivatives of total displacement and thus includes the effects of both mechanical and thermal loading, respectively. The boundary conditions, however, are applied to the mechanical component of the total traction only. This component is explicitly obtained by adding and subtracting the thermal component given by the quantity $((3\lambda+2\mu)\alpha T\{n\})$ from the right-hand side of eqn (9). Rearranging, we get

$$[F]\{u\} = [G]\{t^{(e)}\} + [F]\{u^p\} - [G]\{\bar{t}^p\} \tag{10}$$

where

$$\{t^{(e)}\} = \{t\} - (3\lambda+2\mu)\alpha T\{n\} \tag{11}$$

$$\{\bar{t}^p\} = \{t^p\} - (3\lambda+2\mu)\alpha T\{n\}. \tag{12}$$

In eqn (11), $\{t^{(e)}\}$ is the externally applied traction and the specified traction boundary conditions are applied to this quantity, $\{n\}$ is the outward normal vector at the boundary collocation point. The matrix $\{\bar{t}^p\}$ is referred to here as the modified particular integral vector. It is noted that the vectors $\{u^p\}$ and $\{t^p\}$ may include the contributions due to the various types of body forces acting on the object, such as gravitational, centrifugal, and thermal.

REGRESSION MODEL FOR TEMPERATURE DISTRIBUTIONS

A procedure for obtaining the particular integrals for thermoelastic response is developed next. The following discussion holds for any non-conservative body force with a general distribution over the domain of the body. Only the case of thermal body force is discussed in detail.

It is not generally possible to obtain the particular solutions $\{u^p\}$ and $\{t^p\}$ for a general distribution of body forces. To enable the determination of particular integral solutions for these body forces, their distribution is first represented in a form for which it is possible to obtain such particular integral solutions using the conventional methods, such as the method of undetermined coefficients. In the present study, the distribution of body forces is represented using a linear regression model.

A simple regression model for a system defined by a single variable can be written as

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{13}$$

where x_i are the values of the independent variables, for example, geometric coordinates; y_i are the values of the response variables, for example, temperature; β_0 and β_1 are the regression coefficients; and ε_i is an error term with mean $E(\varepsilon_i) = 0$.

For the present case, it is required to determine the regression coefficients β_0 and β_1 based on the quantities (x_i, y_i) available at N data points. To find "good" estimators of the regression parameters, the method of least squares is employed for which β_0 and β_1 have the values b_0 and b_1, respectively, that minimize the quantity Q given by

$$Q = \sum_{i=1}^{N} (y_i - \beta_0 - \beta_1 x_i)^2. \tag{14}$$

To minimize Q, its partial derivatives with respect to the coefficients to be determined are set equal to zero. Thus, b_0 and b_1 can be obtained from the simultaneous equations

$$\sum_{i=1}^{N} y_i - Nb_0 - b_1 \sum_{i=1}^{N} x_i = 0$$
$$\sum_{i=1}^{N} x_i y_i - b_0 \sum_{i=1}^{N} x_i - b_1 \sum_{i=1}^{N} x_i^2 = 0. \tag{15}$$

The above procedure describes a simple regression model for one independent variable. A similar procedure is applied for more than one independent variable. For such cases not all terms of the regression model are retained. The decision for retaining or deleting a term corresponding to a variable or product of variables may be based on some criterion, such as, (a) the sum of squares of error (SSE) reduction criteria, (b) coefficients of partial correlation criteria, or (c) the F^* statistics criteria. These criteria are described in the textbooks on statistics, for example, Neter *et al.* (1985).

In the present study, a stepwise regression scheme in conjunction with the F^* statistics criteria was employed in obtaining the regression model. The stepwise regression scheme proceeds by adding a new term to the regression model and thereafter applying F^* criteria, term by term, to determine if the existing terms should be retained or deleted. This procedure thus results in a robust representation of the body force field being modeled. The regression models were obtained in this study using the subroutine library SPSSX (1983).

PARTICULAR INTEGRALS FOR REGRESSION MODELS

The particular integral expressions for terms up to the third-order in the regression model are presented below. Corresponding expressions for terms of order higher than three can be similarly developed. Consider the regression model given by the polynomial up to the third-order as

$$T = b_1 + b_2 x + b_3 y + b_4 x^2 + b_5 y^2 + b_6 xy + b_7 x^3 + b_8 y^3 + b_9 x^2 y + b_{10} xy^2. \tag{16}$$

The particular integral expressions corresponding to the regression model in eqn (16) are given as

$$u_x^p = -\frac{c_1 x^2 + c_2 y^2}{8.0\mu c_3} b_2 + \frac{c_2 xy}{4.0\mu c_3} b_3 + \frac{\mu xy^2}{c_1 c_2} b_4 - \frac{c_3 xy^2}{c_1 c_2} b_5 - \frac{y^3}{6.0\mu} b_6 - \frac{x^4}{4.0 c_1} b_7 - \left(\frac{x^3 y}{6.0\mu} + \frac{xy^3}{6.0 c_1}\right) b_9 - \left(\frac{y^4}{12.0\mu} - \frac{c_3 x^2 y^2}{4.0\mu c_1}\right) b_{10} \tag{17}$$

$$u_y^p = \frac{c_2 xy}{4.0\mu c_3} b_2 - \frac{c_2 x^2 + c_1 y^2}{8.0\mu c_3} b_3 - \frac{c_3 yx^2}{c_1 c_2} b_4 + \frac{\mu yx^2}{c_1 c_2} b_5 - \frac{x^3}{6.0\mu} b_6 - \frac{y^4}{4.0 c_1} b_8 - \left(\frac{x^4}{12.0\mu} - \frac{c_3 x^2 y^2}{4.0\mu c_1}\right) b_9 - \left(\frac{y^3 x}{6.0\mu} + \frac{x^3 y}{6.0 c_1}\right) b_{10} \tag{18}$$

where

$$c_1 = \lambda + 2\mu, \quad c_2 = \lambda, \quad c_3 = \lambda + \mu.$$

For the terms in eqn (16) that are not selected by the stepwise regression procedure explained before, the corresponding coefficients are set equal to zero in eqns (16)–(18). To obtain the particular integrals for tractions, the strains corresponding to the displacement particular integrals in eqns (17) and (18) are first determined from the relation

$$\varepsilon_{ij}^p = \frac{1}{2}\left(\frac{\partial u_i^p}{\partial x_j} + \frac{\partial u_j^p}{\partial x_i}\right). \tag{19}$$

Using Hooke's Law, the stresses can then be obtained as

$$\sigma_{ij}^p = \lambda \delta_{ij} \varepsilon_{kk}^p + 2\mu \varepsilon_{ij}^p. \tag{20}$$

Finally, the traction solutions are obtained using the relation

$$t_i^p = \sigma_{ij}^p n_j \tag{21}$$

where n_j is the outward normal.

The particular integral expressions for the axisymmetric case can be similarly determined. For example, for a regression model representing the radial distribution of temperatures in the form

$$T = \sum_{K=0}^{n} b_K r^K \tag{22}$$

the particular integral expressions for displacements may be written as

$$u_r^p = \frac{(1-2\nu)}{2\mu(1-\nu)} \sum_{K=0}^{n} \frac{b_K r^{(K+2)}}{(K+3)}$$
$$u_z^p = 0. \tag{23}$$

The traction particular integrals are derived by a procedure similar to the procedure explained above for the 2-D case.

APPROPRIATE CHOICE OF PARTICULAR INTEGRAL SOLUTION

Any function u_i^p that satisfies eqn (6) is considered to be a valid particular integral solution. It is known that this function is not unique. A proper care must be exercised in the selection of particular integrals for computational purposes. The adverse effect of an improper selection is explained by considering a particular integral solution which can be written as

$$u_i^p = \bar{u}_i^p + \bar{u}_i^c$$
$$t_i^p = \bar{t}_i^p + \bar{t}_i^c \tag{24}$$

where $\bar{u}_i^p$ and $\bar{u}_i^c$ are such that $\bar{u}_i^p$ satisfies eqn (6) and $\zeta(\bar{u}_i^c) = 0$. Thus $\zeta(\bar{u}_i^p + \bar{u}_i^c)$ also satisfies eqn (6) and the expressions in eqn (24) constitute a valid particular integral solution. Substituting eqn (24) into eqn (9) gives

$$[F]\{u\} = [G]\{t\} + [F](\{\bar{u}^p\} + \{\bar{u}^c\}) - [G](\{\bar{t}^p\} + \{\bar{t}^c\}). \tag{25}$$

Rearranging eqn (25), we get

$$[F]\{u\} = [G]\{t\} + [F]\{\bar{u}^p\} - [G]\{\bar{t}^p\} + ([F]\{\bar{u}^c\} - [G]\{\bar{t}^c\}). \quad (26)$$

The last term within parentheses on the right-hand side of eqn (26) should vanish since it involves the complementary system as seen from eqn (8). However, since $[F]$ and $[G]$ are obtained numerically, this term will not be exactly equal to zero in a boundary element analysis and will provide an erroneous contribution to the right-hand side of eqn (26). This will then lead to an error in the solution of response variables obtained from eqn (26).

DATA POINTS FOR REGRESSION MODEL

To obtain a regression model representative of the distribution of temperatures in the domain, the model should be obtained based on a sufficient number of data points. These data points may lie on the boundary or within the domain. The computation of temperatures at any location in the domain of the object, however, requires additional computation in the BEM approach. For the case of steady state thermoelasticity, since the extreme temperatures occur on the boundary, it is sufficient to use data points only on the boundary to obtain the regression model. The transient thermoelasticity problems may also be treated by considering only the boundary data points for the regression model. This may not, however, provide a very accurate response especially for cases when high temperature gradients exist in the domain. To obtain an improved response, some internal points may be used in addition to the boundary points for the regression model. It is noted that the inclusion of these internal points does not add to the number of unknowns but serves merely to improve the quality of the regression model. In case of multi-zone BEM analysis, a separate regression model may be developed for each zone to provide a better representation of the distribution of temperatures in the domain. For large domains it is recommended that these domains be broken up into zones as the regression polynomial representation of temperature field in a large domain may not lead to very accurate results.

NUMERICAL RESULTS

The boundary integral eqn (10) is solved to obtain the thermoelastic response. The usual procedure of applying the known boundary conditions, assembling the unknown boundary quantities on the left-hand side and the known quantities on the right, and solving the resulting system of equations is followed (Banerjee and Butterfield, 1981). The above formulation is applied to obtain numerical results for both steady state and transient responses. The computations reported here were carried out on a RIDGE 3200 computer system. The two-dimensional steady state thermoelastic analysis by regression particular integrals is evaluated for a thick strip with a central hole under plane strain conditions. The axisymmetric steady state and transient thermoelastic response is examined for a thick cylinder under plane strain conditions.

Steady state response of a strip with a hole

A rectangular strip with a circular hole subjected to a harmonic temperature distribution was examined. A half symmetry model of the strip along with the geometric data are shown in Fig. 1a. The material data used were: modulus of elasticity, $E = 1000$ psi; Poisson's ratio, $\nu = 0.3$; coefficient of thermal expansion, $\alpha = 0.02$ in/in/°F. The harmonic temperature distribution on the strip was given by

$$T(x,y) = x^2 - y^2 - 4.0. \quad (27)$$

The half strip was modeled using 17 continuous quadratic boundary elements as shown in Fig. 1a. In this figure, all boundary elements are of equal length. The top and bottom surfaces ($y = 0$ and $y = 3$ in) are both free, the left vertical face ($x = 0$) is completely fixed, and the vertical sides of the right face ($x = 4$ in) are supported on rollers in the x-direction to simulate the symmetry condition. The results were obtained for three cases. Case I: Using

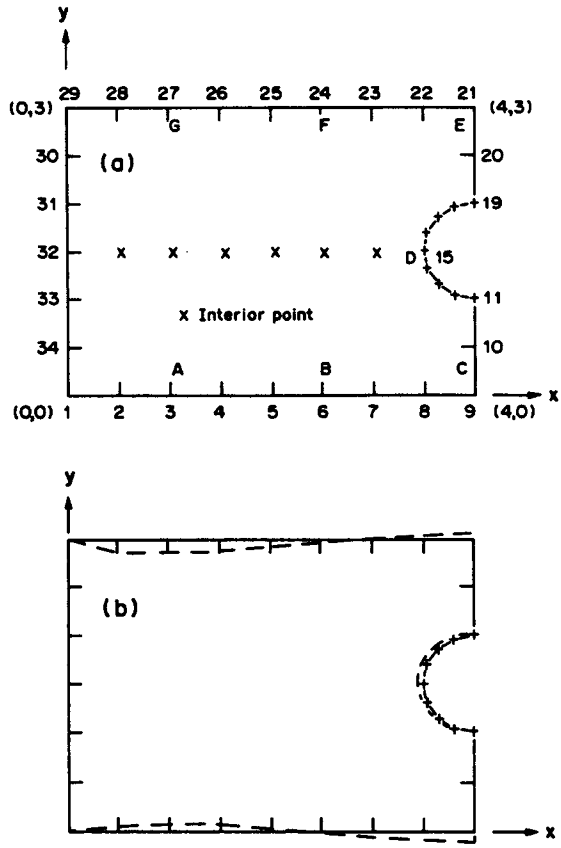

Fig. 1. Thick strip with a central hole (a) boundary element mesh, (b) deflected profile due to prescribed temperature distribution.

the surface integral formulation for thermal body forces caused by harmonic temperature distribution due to Cruse *et al.* (1977); Case II: Using the particular integral formulation based on exact temperature distribution given by eqn (27); and Case III: Using the particular integral formulation based on the regression models developed in this paper for representing the temperature distribution. It is noted that for Case III the regression model produced the exact distribution given by eqn (27) using only the boundary point information. However, these models were constrained so that the terms with x^2 and y^2 both have a coefficient of zero. Two different regression models were generated. The first model [Case III (a)] was based solely on the boundary points and is given as

$$T(x,y) = -4.007028+0.802753x-0.625604y+0.374589x^3 -0.510694y^3-0.043634x^4+0.082170y^4. \quad (28)$$

The second model [Case III (b)] was based on the boundary points as well as six internal points as shown in Fig. 1. This model is given as

$$T(x,y) = -4.019011+0.799393x-0.641360y+0.380352x^3 -0.504413y^3-0.044988x^4+0.080640y^4. \quad (29)$$

The results for all these cases are shown in Tables 1 and 2. It is seen that for an exact representation as in Case II, the particular integral technique produces very accurate results. For approximate representations of the temperature field such as those using the regression models [Case III (a) and Case III (b)] an improved accuracy is obtained as the number of data points used to create the regression model are increased. Good agreement is seen for

Table 1. Comparison of displacements in two dimensional response using surface integral approach and regression particular integral approach for a thick strip with a central circular hole

Node	Case I U_x	Case I U_y	Case II U_x	Case II U_y	Case III (a) U_x	Case III (a) U_y	Case III (b) U_x	Case III (b) U_y
A	−1.492	1.486	−1.491	1.487	−1.442	1.479	−1.448	1.485
B	−2.923	−0.201	−2.922	−0.200	−2.918	−0.142	−2.927	−0.144
C	0.000	−2.552	0.000	−2.552	0.000	−2.536	0.000	−2.539
D	−1.287	0.554	−1.280	0.556	−1.282	0.549	−1.284	0.549
E	0.000	1.023	0.000	1.023	0.000	1.014	0.000	1.017
F	−3.062	−1.318	−3.061	−1.318	−3.056	−1.371	−3.065	−1.368
G	−1.686	−3.183	−1.686	−3.183	−1.636	−3.168	−1.641	−3.173

Displacement (in)

Table 2. Comparison of stresses in two-dimensional response using surface integral approach and regression particular integral approach for a thick strip with a central circular hole

Y-coord. (in)	Stress at fixed end, σ_{xx} (psi) Case I	Case II	Case III (a)	Case III (b)
0.00	164.11	164.21	163.07	163.80
0.50	−52.15	−52.13	−49.32	−49.38
1.00	−1.63	−1.60	−1.62	0.02
1.50	10.55	10.58	9.16	9.29
2.00	78.29	78.33	79.05	79.12
2.50	56.06	56.13	58.08	57.89
3.00	636.05	636.21	634.25	634.85

both the displacements and stresses due to thermoelastic behavior. In Table 2 which shows a comparison of stresses for various cases, a value of 0.02 psi corresponding to $y = 1$ in for Case III (b) does not represent a disagreement with the results obtained for the other cases or a bad convergence with an increase in the number of data points. This is because for all the cases at this location the stress value nearly vanishes compared to the maximum stress at $y = 3$ in. A plot showing the deformed shape of the strip is also shown in Fig. 1b. It is noted from this deformed profile that the extent of deformation in the strip is severe and that the present formulation can predict such deformations with reasonable accuracy.

Steady state response of a cylinder

The cylinder has an inner radius R1 (=3 in) and an outer radius R2 (= 6 in). The material data used was: modulus of elasticity, $E = 1000$ psi; Poisson's ratio, $\nu = 0.3$; coefficient of thermal expansion, $\alpha = 0.02$/in/in°F; density, $\rho = 0.283$ lb/in^3; specific heat = 0.8×10^{-3} Btu/in s °F. A radial section of the cylinder was modeled using eight axisymmetric boundary elements. The geometry and mesh distribution for the axisymmetric model are shown in Fig. 2. All elements in this figure are of equal length. In the present study, continuous quadratic boundary elements were employed.

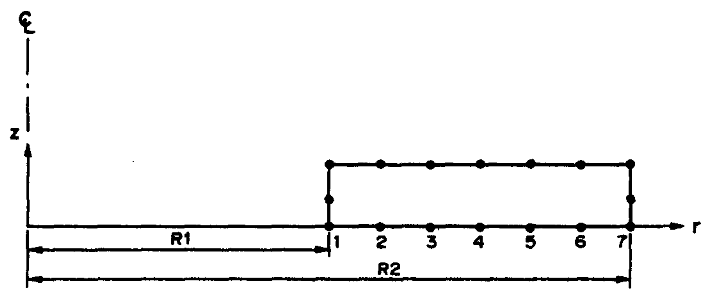

Fig. 2. Axisymmetric boundary element mesh for a thick cylinder.

The inner face of the circular cylinder is subjected to a temperature of 5°F while the outer face is subjected to a temperature of 3°F. The steady state temperature distribution in the cylinder due to these conditions is given by

$$T(r) = 5.0 - \frac{2.0 \ln r_i}{\ln (r_i/r_o)} + \frac{2.0 \ln r}{\ln (r_i/r_o)} \tag{30}$$

where r is the radial distance, r_i is the inner radius, and r_o is the outer radius. The thermoelastic stresses due to this distribution are obtained by an axisymmetric analysis. These results are also compared with the analytical solution by Boley and Weiner (1960) to assess the effectiveness of the present formulation.

Two different cases with regression models of different orders were obtained for this study. Both these models were based on the temperatures at 16 boundary points corresponding to the BEM mesh. The first case uses a regression model with a linear temperature distribution and the second case uses a regression model with a quadratic temperature distribution. The regression models were obtained using stepwise regression procedure and are given as

Model 1, Linear distribution:

$$T = 6.878536 - 0.660870r \tag{31}$$

Model 2, Quadratic distribution:

$$T = 8.337613 - 1.343036r + 0.075796r^2. \tag{32}$$

The thermoelastic response results for both these cases are shown in Tables 3 and 4. The analytical results for this case are also shown in Tables 3 and 4 for comparison. A good agreement of results is obtained for both the linear and the quadratic temperature representations.

Table 3. Radial displacements in axisymmetric thermoelastic response for a thick cylinder under a steady state temperature distribution

	Radial displacement (U_r) (in)		
Node	Analytical	Model 1	Model 2
1	0.29453	0.29599	0.29451
2	0.36069	0.36137	0.36072
3	0.41771	0.41917	0.41791
4	0.46783	0.47055	0.46807
5	0.51249	0.51619	0.51260
6	0.55267	0.55656	0.55261
7	0.58906	0.59194	0.58898

Table 4. Axial stresses and hoop stresses in axisymmetric thermoelastic response for a thick cylinder under a steady state temperature distribution

	Axial stress (σ_z) (psi)			Hoop stress (σ_θ) (psi)		
Node	Analytical	Model 1	Mode 2	Analytical	Model 1	Model 2
1	−110.49	−107.19	−110.02	−34.971	−31.788	−35.235
2	−97.78	−97.98	−98.16	−18.510	−18.363	−18.577
3	−86.77	−88.36	−86.87	−6.446	−8.030	−6.968
4	−77.06	−79.06	−77.07	2.907	1.466	3.095
5	−68.38	−69.55	−68.07	10.466	9.756	10.533
6	−60.52	−60.17	−60.33	16.775	17.706	17.055
7	−53.34	−50.69	−53.55	22.172	25.155	21.844

Transient response of a cylinder

The transient uncoupled thermoelastic response of the above cylinder was obtained using the present axisymmetric BEM formulation. The entire cylinder was initially assumed to be at a temperature of 0°F. The inner and the outer faces of the cylinder were then suddenly raised to a temperature of 5°F and 3°F, respectively. A finite element solution was first obtained for this problem using the commercial finite element code ABAQUS (Hibbitt *et al.*, 1987). The cylinder was discretized using six, 8-noded, axisymmetric finite elements. For the thermal analysis, the element DCAX8 was used, and for the thermoelastic analysis the element CAX8 was used. The temperature distributions in the cylinder at different time instants obtained from the finite element analysis are shown in Fig. 3. These temperature distributions were used in the present study to generate the regression models for thermoelastic BEM analysis. Different regression models were generated at different time instants using the stepwise regression procedure. The regression models were obtained based only on the 16 boundary points corresponding to the boundary element nodes and are given as

Time, $t = 21.1$ s

$$T = 44.807698 - 21.320794r + 2.782188r^2 - 0.010830r^4 \tag{33}$$

Time, $t = 56.9$ s

$$T = 25.978351 - 10.954468r + 1.366600r^2 - 0.004969r^4 \tag{34}$$

Time, $t = 102.0$ s

$$T = 16.414416 - 5.771885r + 0.678490r^2 - 0.002473r^4 \tag{35}$$

Time, $t = 656.0$ s

$$T = 9.110988 - 1.827841r + 0.158292r^2 - 6.50443 \times 10^{-4} r^4. \tag{36}$$

The transient thermoelastic results from the present analysis are shown in Figs 4

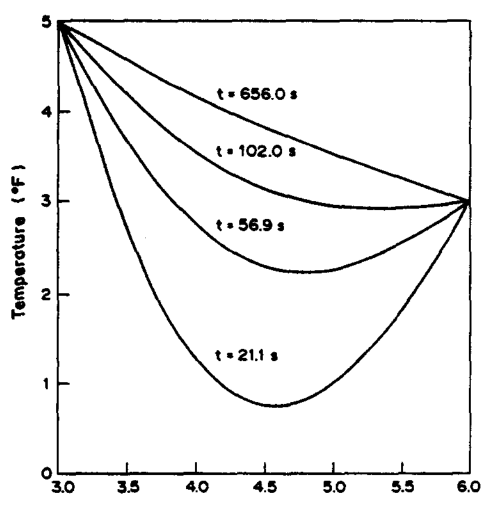

Fig. 3. Temperature distribution at various time instants for a thick cylinder.

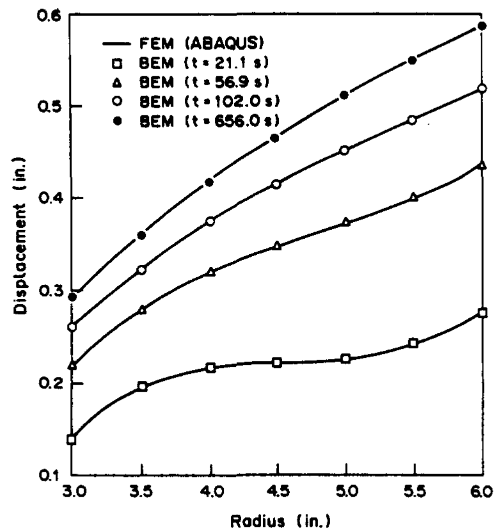

Fig. 4. Radial displacements at various time instants for a thick cylinder.

through 6 along with the results obtained using finite element analysis. A good agreement of results is seen. An improved agreement of the thermoelastic results during the early transient response may be obtained by using a few internal points also, since high temperature gradients exist in the domain during this stage of the response.

CONCLUDING REMARKS

The distribution of temperature within the domain of a body is represented using a regression model. The model is based on the temperature data at the boundary element nodes and a few interior points. A stepwise regression technique that ensures the selection of only the most effective terms is used to obtain the regression model. This model uses lower order polynomials and fewer terms to effectively represent the temperature distribution in a domain. Particular integrals are developed for the regression model obtained to account for the thermal body forces. The present technique avoids the need for subdividing the domain into cells for computing volume integrals for the treatment of non-uniform body forces. In this paper the non-uniform body force field is represented using a regression model which provides an approximate representation of this field. For a large domain, this

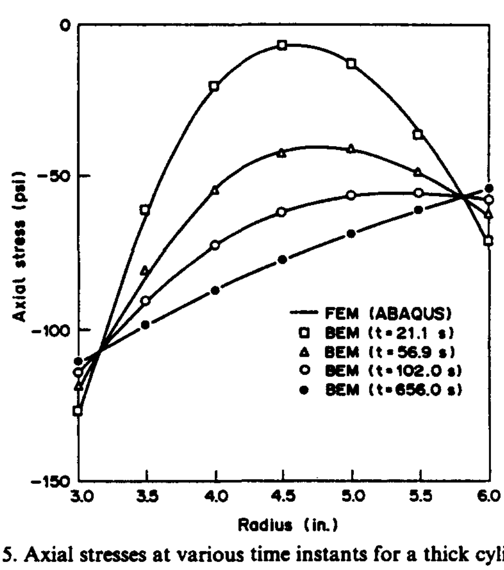

Fig. 5. Axial stresses at various time instants for a thick cylinder.

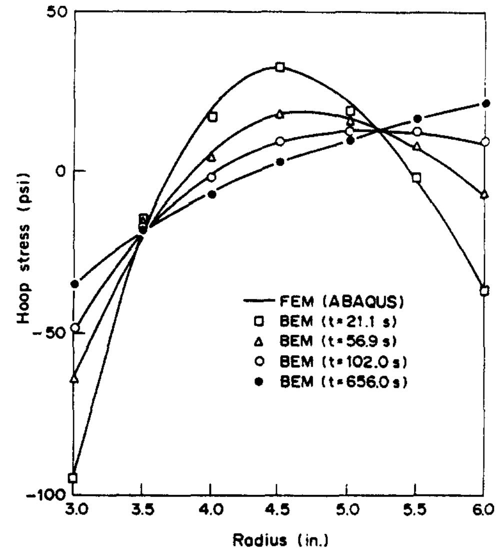

Fig. 6. Hoop stresses at various time instants for a thick cylinder.

representation will provide only an approximate response but at the same time will highlight the high stress regions. In these situations it is recommended that, after first peforming the "boundary data only" analysis, a more extensive analysis be carried out by subdividing the domain into multiple regions and developing the regression model for each region separately. This approach is applied to the analysis of steady state and transient thermoelastic responses of two-dimensional and axisymmetric boundary element problems, respectively. Numerical examples are presented to demonstrate the effectiveness of the present technique.

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